Assignment 4

Due Friday, 7/25/2014

Remember to show your work for credit!

Problem 1

Solve each of the following initial value problems.

(a) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$.

(b) $25y'' - 20y' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$.

(c) $y'' + 2y' + 2y = 0$, $y(\pi/4) = 2$, $y'(\pi/4) = -2$.

(d) $9y'' + 6y' + 82y = 0$, $y(0) = -1$, $y'(0) = 2$.

Problem 2

Use the method of undetermined coefficients to find the general solution for each of the following differential equations.

(a) $y'' + 2y' + 5y = 3\sin(2t)$.

(b) $y'' - 2y' - 3y = 3e^{2t}$.

(c) $y'' + 9y = t^2e^{3t} + 6$.

Problem 3

Use variation of parameters to find the general solution to the following differential equation:

$$x^2y'' - 3xy' + 4y = x^2 \ln x,$$

where $x > 0$. Hint: the solution to the homogeneous problem is

$$y_H = C_1x^2 + C_2x^2 \ln x.$$

Problem 4

Consider a spring-mass system with mass m, friction γ, spring constant k and external forces $F_0 \sin(\omega t)$. Suppose that $0 < \gamma < \sqrt{4mk}$. This system is modeled by the equation

$$my'' + \gamma y' + ky = F_0 \sin(\omega t).$$

(a) Find the general solution to this equation.

(b) What happens to the homogeneous solution when t goes to infinity? What happens to the particular solution when t goes to infinity?

(c) Fix $m = k = F_0 = 1$ and $\gamma = .5$. Graph the particular solution for the following values of ω: $\omega = 1/4$, $1/2$, $3/4$, $7/8$, 1, $9/8$, $5/4$ and $3/2$. For each value of ω, estimate the amplitude/maximum height of the particular solution (or calculate it exactly). (**Update:** Since these graphs are all very similar, you only need to include one in the write-up. You do, however, need to know the amplitude for each so that you can do the next part.) Using these estimates, plot the maximum amplitude as a function of ω.