Assignment 5

Due 8/1/2014

Problem 1

Use the ratio test to find the interval of convergence for the following series using the ratio test:

(a) $\sum_{n=0}^{\infty}(x+4)^n$.

(b) $\sum_{n=0}^{\infty}\frac{nx^n}{3^n}$.

(c) $\sum_{n=1}^{\infty}\frac{n!x^n}{n^n}$.

Problem 2

Consider the function

$$J_0(x) = 1 + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}.$$

(a) Show that the interval of convergence of J_0 is $-\infty < x < \infty$.

(b) Show that the series satisfies the differential equation

$$x^2y'' + xy' + x^2y = 0.$$

This function is called the Bessel function of the first kind of order zero.

Problem 3

Solve the following differential equations by finding a series solution about $x_0 = 0$. Find the recursion relation and the first four terms in each of the two linearly independent solutions.

(a) $y'' - xy' - y = 0$.

(b) $2y'' + xy' + 3y = 0$.

Problem 4

Solve the following Euler equation initial value problems for $x > 0$. Plot the solution.

(a) $2x^2y'' + xy' - 3y = 0$, $y(1) = 1$, $y'(1) = 4$.

(b) $x^2y'' + 3xy' + 5y = 0$, $y(1) = 1$, $y'(1) = -1$.

Problem 5

The Legendre equation is given by

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0.$$

This equation appears in many physical applications (such as models of electromagnetism and fluids).

(a) Find the solution to the Legendre equation using a power series about $x_0 = 0$. Find the recursion relation and the first four terms in each of the two linearly independent solutions.

(b) Show that for $\alpha = 1$, $\alpha = 2$ and $\alpha = 3$, one of the solutions is actually a polynomial. (That is, it only has finitely many terms.)