

## Assignment 6 Due 8/8/2014

### Problem 1

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Derive the laplace transform  $F(s)$  of each of the following functions. (The final answers are in the notes – you need to show your derivation.)

- (a)  $f(t) = \cos(at)$ .
- (b)  $f(t) = t$ .
- (c)  $f(t) = t^2$ .
- (d)  $f(t) = t^n$  where  $n$  is a whole number.

### Problem 2

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Find the inverse laplace transform  $f(t)$  of each of the following functions.

- (a)  $F(s) = \frac{2}{s^2 + 3s - 10}$ .
- (b)  $F(s) = \frac{e^{-2s}}{s^2 + s - 2}$ .
- (c)  $F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$ .

### Problem 3

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Consider the function

$$y(t) = \begin{cases} 0 & t \leq 1, \\ t - 1 & 1 < t \leq 2, \\ 1 & t > 2. \end{cases}$$

- (a) Graph  $y(t)$ .
- (b) Write a one-line formula for  $y$  using the Heaviside function  $u(t)$ .
- (c) Find the Laplace transform of  $y(t)$ .

## Problem 4

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Solve the following initial value problems using the method of Laplace transforms.

(a)  $y'' + y = \sin(2t)$ ,  $y(0) = 2$ ,  $y'(0) = 1$ .

(b)  $y'' + 3y' + 2y = u(t - 2)$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

(c)  $y'' + 2y' + y = 4e^{-t}$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

(d)  $y'' + y' + 5y/4 = u(t) \sin(t) - u(t - \pi) \sin(t)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .